By analogy, we have for the opposite event $\neg x$:

$$p(\neg x \mid z_{1:t}) \quad = \quad \frac{p(\neg x \mid z_t)\, p(z_t)\, p(\neg x \mid z_{1:t-1})}{p(\neg x)\, p(z_t \mid z_{1:t-1})} \tag{4.18}$$

Dividing (4.17) by (4.18) leads to cancellation of various difficult-to-calculate probabilities:

$$\begin{aligned} \frac{p(x \mid z_{1:t})}{p(\neg x \mid z_{1:t})} &= \frac{p(x \mid z_t)}{p(\neg x \mid z_t)} \frac{p(x \mid z_{1:t-1})}{p(\neg x \mid z_{1:t-1})} \frac{p(\neg x)}{p(x)} \\ &= \frac{p(x \mid z_t)}{1 - p(x \mid z_t)} \frac{p(x \mid z_{1:t-1})}{1 - p(x \mid z_{1:t-1})} \frac{1 - p(x)}{p(x)} \end{aligned} \tag{4.19}$$

We denote the log odds ratio of the belief $bel_t(x)$ by $l_t(x)$. The log odds belief at time t is given by the logarithm of (4.19).

$$\begin{aligned} l_t(x) &= \log \frac{p(x \mid z_t)}{1 - p(x \mid z_t)} + \log \frac{p(x \mid z_{1:t-1})}{1 - p(x \mid z_{1:t-1})} + \log \frac{1 - p(x)}{p(x)} \\ &= \log \frac{p(x \mid z_t)}{1 - p(x \mid z_t)} - \log \frac{p(x)}{1 - p(x)} + l_{t-1}(x) \end{aligned} \tag{4.20}$$

Here $p(x)$ is the *prior* probability of the state x. As in (4.20), each measurement update involves the addition of the prior (in log odds form). The prior also defines the log odds of the initial belief before processing any sensor measurement:

$$l_0(x) \quad = \quad \log \frac{p(x)}{1-p(x)} \tag{4.21}$$

4.3 The Particle Filter

4.3.1 Basic Algorithm

The *particle filter* is an alternative nonparametric implementation of the Bayes filter. Just like histogram filters, particle filters approximate the posterior by a finite number of parameters. However, they differ in the way these parameters are generated, and in which they populate the state space. The key idea of the particle filter is to represent the posterior $bel(x_t)$ by a set of random state samples drawn from this posterior. Figure 4.3 illustrates this idea for a Gaussian. Instead of representing the distribution by a parametric form—which would have been the exponential function that defines the density of a normal distribution—particle filters represent a distribution by a set of samples drawn from this distribution. Such a representation is approximate,