ones, but at the expense of increased computational complexity.

As we already discussed, the discrete Bayes filter assigns to each region $\mathbf{x}_{k,t}$ a probability, $p_{k,t}$. Within each region, the discrete Bayes filter carries no further information on the belief distribution. Thus, the posterior becomes a piecewise constant PDF, which assigns a uniform probability to each state x_t within each region $\mathbf{x}_{k,t}$:

$$p(x_t) \quad = \quad \frac{p_{k,t}}{|\mathbf{x}_{k,t}|} \tag{4.2}$$

Here $|\mathbf{x}_{k,t}|$ is the volume of the region $\mathbf{x}_{k,t}$.

If the state space is truly discrete, the conditional probabilities $p(\mathbf{x}_{k,t} \mid u_t, \mathbf{x}_{i,t-1})$ and $p(z_t \mid \mathbf{x}_{k,t})$ are well-defined, and the algorithm can be implemented as stated. In continuous state spaces, one is usually given the densities $p(x_t \mid u_t, x_{t-1})$ and $p(z_t \mid x_t)$, which are defined for individual states (and not for regions in state space). For cases where each region $\mathbf{x}_{k,t}$ is small and of the same size, these densities are usually approximated by substituting $\mathbf{x}_{k,t}$ by a representative of this region. For example, we might simply "probe" using the mean state in $\mathbf{x}_{k,t}$

$$\hat{x}_{k,t} \quad = \quad |\mathbf{x}_{k,t}|^{-1} \int_{\mathbf{x}_{k,t}} x_t \; dx_t \tag{4.3}$$

One then simply replaces

$$p(z_t \mid \mathbf{x}_{k,t}) \quad \approx \quad p(z_t \mid \hat{x}_{k,t}) \tag{4.4}$$

$$p(\mathbf{x}_{k,t} \mid u_t, \mathbf{x}_{i,t-1}) \quad \approx \quad \eta \; |\mathbf{x}_{k,t}| \; p(\hat{x}_{k,t} \mid u_t, \hat{x}_{i,t-1}) \tag{4.5}$$

These approximations are the result of the piecewise uniform interpretation of the discrete Bayes filter stated in (4.2), and a Taylor-approximation analogous to the one used by EKFs.

4.1.3 Mathematical Derivation of the Histogram Approximation

To see that (4.4) is a reasonable approximation, we note that $p(z_t \mid \mathbf{x}_{k,t})$ can be expressed as the following integral:

$$\begin{aligned} p(z_t \mid \mathbf{x}_{k,t}) \quad &= \quad \frac{p(z_t, \mathbf{x}_{k,t})}{p(\mathbf{x}_{k,t})} \\ &= \quad \frac{\displaystyle\int_{\mathbf{x}_{k,t}} p(z_t, x_t) \; dx_t}{\displaystyle\int_{\mathbf{x}_{k,t}} p(x_t) \; dx_t} \end{aligned} \tag{4.6}$$