3.4 The Unscented Kalman Filter

The Taylor series expansion applied by the EKF is only one way to linearize the transformation of a Gaussian. Two other approaches have often been found to yield superior results. One is known as *moments matching* (and the resulting filter is known as *assumed density filter*, or *ADF*), in which the linearization is calculated in a way that preserves the true mean and the true covariance of the posterior distribution (which is not the case for EKFs). Another linearization method is applied by the *unscented Kalman filter*, or *UKF*, which performs a stochastic linearization through the use of a weighted statistical linear regression process. We now discuss the UKF algorithm without mathematical derivation. The reader is encouraged to read more details in the literature referenced in the bibliographical remarks.

UNSCENTED KALMAN FILTER

3.4.1 Linearization Via the Unscented Transform

Figure 3.7 illustrates the linearization applied by the UKF, called the *unscented transform*. Instead of approximating the function g by a Taylor series expansion, the UKF deterministically extracts so-called *sigma points* from the Gaussian and passes these through g. In the general case, these sigma points are located at the mean and symmetrically along the main axes of the covariance (two per dimension). For an n-dimensional Gaussian with mean μ and covariance Σ, the resulting $2n+1$ sigma points $\mathcal{X}^{[i]}$ are chosen according to the following rule:

SIGMA POINT

(3.66)
$$
\begin{aligned}
\mathcal{X}^{[0]} &= \mu \\
\mathcal{X}^{[i]} &= \mu + \left(\sqrt{(n+\lambda)\,\Sigma}\right)_i \qquad \text{for } i = 1, \ldots, n \\
\mathcal{X}^{[i]} &= \mu - \left(\sqrt{(n+\lambda)\,\Sigma}\right)_{i-n} \qquad \text{for } i = n+1, \ldots, 2n
\end{aligned}
$$

Here $\lambda = \alpha^2(n+\kappa) - n$, with α and κ being scaling parameters that determine how far the sigma points are spread from the mean. Each sigma point $\mathcal{X}^{[i]}$ has two weights associated with it. One weight, $w_m^{[i]}$, is used when computing the mean, the other weight, $w_c^{[i]}$, is used when recovering the covariance of the Gaussian.

(3.67)
$$
\begin{aligned}
w_m^{[0]} &= \frac{\lambda}{n+\lambda} \\
w_c^{[0]} &= \frac{\lambda}{n+\lambda} + (1 - \alpha^2 + \beta)
\end{aligned}
$$