where the matrix $Q_t^{[k]}$ is defined as follows:

(13.35) $$Q_t^{[k]} = Q_t + H_m \Sigma_{c_t,t-1}^{[k]} H_m^T$$

To see, we note that under out linear approximation the convolution theorem provides us with a closed form for the integral term in (13.27):

(13.36) $$\mathcal{N}(z_t; \hat{z}_t^{[k]} + H_x x_t - H_x \hat{x}_t^{[k]}, Q_t^{[k]})$$

The sampling distribution (13.27) is now given by the product of this normal distribution and the rightmost term in (13.27), the normal $\mathcal{N}(x_t; \hat{x}_t^{[k]}, R_t)$. Written in Gaussian form, we have

(13.37) $$p(x_t \mid x_{1:t-1}^{[k]}, u_{1:t}, z_{1:t}, c_{1:t}) = \eta \exp\left\{-P_t^{[k]}\right\}$$

with

(13.38) $$\begin{aligned} P_t^{[k]} = & \frac{1}{2}\Big[(z_t - \hat{z}_t^{[k]} - H_x x_t + H_x \hat{x}_t^{[k]})^T Q_t^{[k]-1} (z_t - \hat{z}_t^{[k]} - H_x x_t + H_x \hat{x}_t^{[k]}) \\ & +(x_t - \hat{x}_t^{[k]})^T R_t^{-1} (x_t - \hat{x}_t^{[k]})\Big] \end{aligned}$$

This expression is obviously quadratic in our target variable x_t, hence $p(x_t \mid x_{1:t-1}^{[k]}, u_{1:t}, z_{1:t}, c_{1:t})$ is Gaussian. The mean and covariance of this Gaussian are equivalent to the minimum of $P_t^{[k]}$ and its curvature. Those are identified by calculating the first and second derivatives of $P_t^{[k]}$ with respect to x_t:

(13.39) $$\begin{aligned} \frac{\partial P_t^{[k]}}{\partial x_t} &= -H_x^T Q_t^{[k]-1}(z_t - \hat{z}_t^{[k]} - H_x x_t + H_x \hat{x}_t^{[k]}) + R_t^{-1}(x_t - \hat{x}_t^{[k]}) \\ &= (H_x^T Q_t^{[k]-1} H_x + R_t^{-1}) x_t - H_x^T Q_t^{[k]-1}(z_t - \hat{z}_t^{[k]} + H_x \hat{x}_t^{[k]}) - R_t^{-1}\hat{x}_t^{[k]} \end{aligned}$$

(13.40) $$\frac{\partial^2 P_t^{[k]}}{\partial x_t^2} = H_x^T Q_t^{[k]-1} H_x + R_t^{-1}$$

The covariance $\Sigma_{x_t}^{[k]}$ of the sampling distribution is now obtained by the inverse of the second derivative

(13.41) $$\Sigma_{x_t}^{[k]} = \left[H_x^T Q_t^{[k]-1} H_x + R_t^{-1}\right]^{-1}$$

The mean $\mu_{x_t}^{[k]}$ of the sample distribution is obtained by setting the first derivative (13.39) to zero. This gives us:

(13.42) $$\begin{aligned} \mu_{x_t}^{[k]} &= \Sigma_{x_t}^{[k]}\left[H_x^T Q_t^{[k]-1}(z_t - \hat{z}_t^{[k]} + H_x \hat{x}_t^{[k]}) + R_t^{-1}\hat{x}_t^{[k]}\right] \\ &= \Sigma_{x_t}^{[k]} H_x^T Q_t^{[k]-1}(z_t - \hat{z}_t^{[k]}) + \Sigma_{x_t}^{[k]}\left[H_x^T Q_t^{[k]-1} H_x + R_t^{-1}\right]\hat{x}_t^{[k]} \\ &= \Sigma_{x_t}^{[k]} H_x^T Q_t^{[k]-1}(z_t - \hat{z}_t^{[k]}) + \hat{x}_t^{[k]} \end{aligned}$$