$$
\begin{aligned}
&\overset{\text{Markov}}{=} \quad \eta^{[k]}\ p(z_t \mid x_t, x_{1:t-1}^{[k]}, u_{1:t}, z_{1:t-1}, c_{1:t}); p(x_t \mid x_{t-1}^{[k]}, u_t) \\
&= \quad \eta^{[k]} \int p(z_t \mid m_{c_t}, x_t, x_{1:t-1}^{[k]}, u_{1:t}, z_{1:t-1}, c_{1:t}) \\
&\qquad p(m_{c_t} \mid x_t, x_{1:t-1}^{[k]}, u_{1:t}, z_{1:t-1}, c_{1:t})\ dm_{c_t}\ p(x_t \mid x_{t-1}^{[k]}, u_t) \\
&\overset{\text{Markov}}{=} \quad \eta^{[k]} \int \underbrace{p(z_t \mid m_{c_t}, x_t, c_t)}_{\sim \mathcal{N}(z_t; h(m_{c_t}, x_t), Q_t)} \quad \underbrace{p(m_{c_t} \mid x_{1:t-1}^{[k]}, z_{1:t-1}, c_{1:t-1})}_{\sim \mathcal{N}(m_{c_t}; \mu_{c_t,t-1}^{[k]}, \Sigma_{c_t,t-1}^{[k]})}\ dm_{c_t} \\
&\qquad \underbrace{p(x_t \mid x_{t-1}^{[k]}, u_t)}_{\sim \mathcal{N}(x_t; g(x_{t-1}^{[k]}, u_t), R_t)}
\end{aligned}
$$

This expression makes apparent that our sampling distribution is truly the convolution of two Gaussians multiplied by a third. In the general SLAM case, the sampling distribution possesses no closed form from which we could easily sample. The culprit is the function h: If it were linear, this probability would be Gaussian, a fact that shall become more obvious below. In fact, not even the integral in (13.27) possesses a closed form solution. For this reason, sampling from the probability (13.27) is difficult.

This observation motivates the replacement of h by a linear approximation. As common in this book, this approximation is obtained through a first order Taylor expansion, given by the following linear function:

(13.28) $$h(m_{c_t}, x_t) \quad \approx \quad \hat{z}_t^{[k]} + H_m (m_{c_t} - \mu_{c_t,t-1}^{[k]}) + H_x\ (x_t - \hat{x}_t^{[k]})$$

Here we use the following abbreviations:

(13.29) $$\hat{z}_t^{[k]} \quad = \quad h(\mu_{c_t,t-1}^{[k]}, \hat{x}_t^{[k]})$$

(13.30) $$\hat{x}_t^{[k]} \quad = \quad g(x_{t-1}^{[k]}, u_t)$$

The matrices H_m and H_x are the Jacobians of h. They are the derivatives of h with respect to m_{c_t} and x_t, respectively, evaluated at the expected values of their arguments:

(13.31) $$H_m \quad = \quad \nabla_{m_{c_t}} h(m_{c_t}, x_t)\Big|_{x_t=\hat{x}_t^{[k]}; m_{c_t}=\mu_{c_t,t-1}^{[k]}}$$

(13.32) $$H_x \quad = \quad \nabla_{x_t} h(m_{c_t}, x_t)\Big|_{x_t=\hat{x}_t^{[k]}; m_{c_t}=\mu_{c_t,t-1}^{[k]}}$$

Under this approximation, the desired sampling distribution (13.27) is a Gaussian with the following parameters:

(13.33) $$\Sigma_{x_t}^{[k]} \quad = \quad \left[H_x^T Q_t^{[k]-1} H_x + R_t^{-1} \right]^{-1}$$

(13.34) $$\mu_{x_t}^{[k]} \quad = \quad \Sigma_{x_t}^{[k]} H_x^T Q_t^{[k]-1} (z_t - \hat{z}_t^{[k]}) + \hat{x}_t^{[k]}$$