$$= \quad \eta \; p(z_t \mid y_{0:t}, z_{1:t-1}, u_{1:t}, c_{1:t}) \; p(y_{0:t} \mid z_{1:t-1}, u_{1:t}, c_{1:t})$$

where η is the familiar normalizer. The first probability on the right-hand side can be reduced by dropping irrelevant conditioning variables:

$$p(z_t \mid y_{0:t}, z_{1:t-1}, u_{1:t}, c_{1:t}) \quad = \quad p(z_t \mid y_t, c_t) \tag{11.6}$$

Similarly, we can factor the second probability by partitioning $y_{0:t}$ into x_t and $y_{0:t-1}$, and obtain

$$\begin{aligned} & p(y_{0:t} \mid z_{1:t-1}, u_{1:t}, c_{1:t}) \\ & \quad = \quad p(x_t \mid y_{0:t-1}, z_{1:t-1}, u_{1:t}, c_{1:t}) \; p(y_{0:t-1} \mid z_{1:t-1}, u_{1:t}, c_{1:t}) \\ & \quad = \quad p(x_t \mid x_{t-1}, u_t) \; p(y_{0:t-1} \mid z_{1:t-1}, u_{1:t-1}, c_{1:t-1}) \end{aligned} \tag{11.7}$$

Putting these expressions back into (11.5) gives us the recursive definition of the full SLAM posterior:

$$\begin{aligned} & p(y_{0:t} \mid z_{1:t}, u_{1:t}, c_{1:t}) \\ & \quad = \quad \eta \; p(z_t \mid y_t, c_t) \; p(x_t \mid x_{t-1}, u_t) \; p(y_{0:t-1} \mid z_{1:t-1}, u_{1:t-1}, c_{1:t-1}) \end{aligned} \tag{11.8}$$

The closed form expression is obtained through induction over t. Here $p(y_0)$ is the prior over the map m and the initial pose x_0.

$$\begin{aligned} p(y_{0:t} \mid z_{1:t}, u_{1:t}, c_{1:t}) \quad & = \quad \eta \; p(y_0) \; \prod_t p(x_t \mid x_{t-1}, u_t) \; p(z_t \mid y_t, c_t) \\ & = \quad \eta \; p(y_0) \; \prod_t \left[p(x_t \mid x_{t-1}, u_t) \; \prod_i p(z_t^i \mid y_t, c_t^i) \right] \end{aligned} \tag{11.9}$$

Here, as before, z_t^i is the i-th measurement in the measurement vector z_t at time t. The prior $p(y_0)$ factors into two independent priors, $p(x_0)$ and $p(m)$. In SLAM, we usually have no prior knowledge about the map m. We simply replace $p(y_0)$ by $p(x_0)$ and subsume the factor $p(m)$ into the normalizer η.

11.4.2 The Negative Log Posterior

The information form represents probabilities in logarithmic form. The log-SLAM posterior follows directly from the previous equation:

$$\begin{aligned} & \log p(y_{0:t} \mid z_{1:t}, u_{1:t}, c_{1:t}) \\ & \quad = \quad \text{const.} + \log p(x_0) \; + \; \sum_t \left[\log p(x_t \mid x_{t-1}, u_t) \; + \; \sum_i \log p(z_t^i \mid y_t, c_t^i) \right] \end{aligned} \tag{11.10}$$