in line 5 are sparse, which should be exploited when implementing this algorithm. The result of this update are the mean $\bar{\mu}_t$ and the covariance $\bar{\Sigma}_t$ of the estimate at time t after updating the filter with the control u_t, but before integrating the measurement z_t.

The derivation of the measurement update is similar to the one in Chapter 7.4. In particular, we are given the following measurement model

$$z_t^i = \underbrace{\begin{pmatrix} \sqrt{(m_{j,x}-x)^2+(m_{j,y}-y)^2} \\ \text{atan2}(m_{j,y}-y, m_{j,x}-x)-\theta \\ m_{j,s} \end{pmatrix}}_{h(y_t,j)} + \mathcal{N}(0, \underbrace{\begin{pmatrix} \sigma_r & 0 & 0 \\ 0 & \sigma_\phi & 0 \\ 0 & 0 & \sigma_s \end{pmatrix}}_{Q_t}) \tag{10.17}$$

Here x, y, and θ denotes the pose of the robot, i is the index of an individual landmark observation in z_t, and $j = c_t^i$ is the index of the observed landmark at time t. The variable r denotes the range to a landmark, ϕ is the bearing to a landmark, and s the landmark signature; the terms σ_r, σ_ϕ, and σ_s are the corresponding measurement noise covariances.

This expression is approximated by the linear function

$$h(y_t, j) \quad \approx \quad h(\bar{\mu}_t, j) \ + \ H_t^i \ (y_t - \bar{\mu}_t) \tag{10.18}$$

Here H_t^i is the derivative of h with respect to the full state vector y_t. Since h depends only on two elements of that state vector, the robot pose x_t and the location of the j-th landmark m_j, the derivative factors into a low-dimensional Jacobian h_t^i and a matrix $F_{x,j}$, which maps h_t^i into a matrix of the dimension of the full state vector:

$$H_t^i \quad = \quad h_t^i \ F_{x,j} \tag{10.19}$$

Here h_t^i is the Jacobian of the function $h(y_t, j)$ at $\bar{\mu}_t$, calculated with respect to the state variables x_t and m_j:

$$h_t^i = \begin{pmatrix} \frac{\bar{\mu}_{t,x}-\bar{\mu}_{j,x}}{\sqrt{q_t}} & \frac{\bar{\mu}_{t,y}-\bar{\mu}_{j,y}}{\sqrt{q_t}} & 0 & \frac{\bar{\mu}_{j,x}-\bar{\mu}_{t,x}}{\sqrt{q_t}} & \frac{\bar{\mu}_{j,y}-\bar{\mu}_{t,y}}{\sqrt{q_t}} & 0 \\ \frac{\bar{\mu}_{j,y}-\bar{\mu}_{t,y}}{q_t} & \frac{\bar{\mu}_{t,x}-\bar{\mu}_{j,x}}{q_t} & -1 & \frac{\bar{\mu}_{t,y}-\bar{\mu}_{j,y}}{q_t} & \frac{\bar{\mu}_{j,x}-\bar{\mu}_{t,x}}{q_t} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{10.20}$$

The scalar $q_t = (\bar{\mu}_{j,x} - \bar{\mu}_{t,x})^2 + (\bar{\mu}_{j,y} - \bar{\mu}_{t,y})^2$, and as before, $j = c_t^i$ is the landmark that corresponds to the measurement z_t^i. The matrix $F_{x,j}$ is of dimension $6 \times (3N + 3)$. It maps the low-dimensional matrix h_t^i into a matrix