the same equation more compactly:

$$y_t = y_{t-1} + F_x^T \begin{pmatrix} -\frac{v_t}{\omega_t}\sin\theta + \frac{v_t}{\omega_t}\sin(\theta+\omega_t\Delta t) \\ \frac{v_t}{\omega_t}\cos\theta - \frac{v_t}{\omega_t}\cos(\theta+\omega_t\Delta t) \\ \omega_t\Delta t + \gamma_t\Delta t \end{pmatrix} \tag{10.11}$$

Here F_x is a matrix that maps the 3-dimensional state vector into a vector of dimension $3N+3$.

$$F_x = \begin{pmatrix} 1 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & \underbrace{0 \cdots 0}_{3N \text{ columns}} \end{pmatrix} \tag{10.12}$$

The full motion model with noise is then as follows

$$y_t = \underbrace{y_{t-1} + F_x^T \begin{pmatrix} -\frac{v_t}{\omega_t}\sin\theta + \frac{v_t}{\omega_t}\sin(\theta+\omega_t\Delta t) \\ \frac{v_t}{\omega_t}\cos\theta - \frac{v_t}{\omega_t}\cos(\theta+\omega_t\Delta t) \\ \omega\Delta t \end{pmatrix}}_{g(u_t,y_{t-1})} + \mathcal{N}(0, F_x^T\ R_t\ F_x) \tag{10.13}$$

where $F_x^T R_t F_x$ extends the covariance matrix to the dimension of the full state vector squared.

As usual in EKFs, the motion function g is approximated using a first degree Taylor expansion

$$g(u_t, y_{t-1}) \approx g(u_t, \mu_{t-1}) + G_t\ (y_{t-1} - \mu_{t-1}) \tag{10.14}$$

where the Jacobian $G_t = g'(u_t, \mu_{t-1})$ is the derivative of g with respect to y_{t-1} at u_t and μ_{t-1}, as in Equation (7.7).

Obviously, the additive form in (10.13) enables us to decompose this Jacobian into an identity matrix of dimension $(3N+3)\times(3N+3)$ (the derivative of y_{t-1}) plus a low-dimensional Jacobian g_t that characterizes the change of the robot pose:

$$G_t = I + F_x^T\ g_t\ F_x \tag{10.15}$$

with

$$g_t = \begin{pmatrix} 0 & 0 & -\frac{v_t}{\omega_t}\cos\mu_{t-1,\theta} + \frac{v_t}{\omega_t}\cos(\mu_{t-1,\theta}+\omega_t\Delta t) \\ 0 & 0 & -\frac{v_t}{\omega_t}\sin\mu_{t-1,\theta} + \frac{v_t}{\omega_t}\sin(\mu_{t-1,\theta}+\omega_t\Delta t) \\ 0 & 0 & 0 \end{pmatrix} \tag{10.16}$$

Plugging these approximations into the standard EKF algorithm gives us lines 2 through 5 of Table 10.1. Obviously, several of the matrices multiplied