1: **Algorithm KLD_Sampling_MCL(**$\mathcal{X}_{t-1}, u_t, z_t, m, \varepsilon, \delta$**):**
2: $\mathcal{X}_t = \emptyset$
3: $M = 0$, $M_\chi = 0$, $k = 0$
4: *for all* b *in* H *do*
5: $b =$ *empty*
6: *endfor*
7: *do*
8: *draw* i *with probability* $\propto w_{t-1}^{[i]}$
9: $x_t^{[M]} =$ **sample_motion_model**$(u_t, x_{t-1}^{[i]})$
10: $w_t^{[M]} =$ **measurement_model**$(z_t, x_t^{[M]}, m)$
11: $\mathcal{X}_t = \mathcal{X}_t + \langle x_t^{[M]}, w_t^{[M]} \rangle$
12: *if* $x_t^{[M]}$ *falls into empty bin* b *then*
13: $k = k + 1$
14: $b =$ *non-empty*
15: *if* $k > 1$ *then*
16: $M_\chi := \frac{k-1}{2\varepsilon} \left\{ 1 - \frac{2}{9(k-1)} + \sqrt{\frac{2}{9(k-1)}} z_{1-\delta} \right\}^3$
17: *endif*
18: $M = M + 1$
19: *while* $M < M_\chi$ or $M < M_{\chi_{min}}$
20: *return* $\mathcal{X}_t$

Table 8.4 KLD-sampling MCL with adaptive sample set size. The algorithm generates samples until a statistical bound on the approximation error is reached.

in line 16 is satisfied. This bound is based on the "volume" of the state space that is covered by particles. The volume covered by particles is measured by a histogram, or grid, overlayed over the 3-dimensional state space. Each bin in the histogram H is either empty or occupied by at least one particle. Initially, each bin is set to empty (Lines 4 through 6). In line 8, a particle is drawn from the previous sample set. Based on this particle, a new particle is predicted, weighted, and inserted into the new sample set (Lines 9–11, just like in MCL).

Lines 12 through 19 implement the key idea of KLD-sampling. If the newly generated particle falls into an empty bin of the histogram, then the number k of non-empty bins is incremented and the bin is marked as non-empty. Thus, k measures the number of histogram bins filled with at least one particle. This number plays a crucial role in the statistical bound determined in line 16. The quantity M_χ gives the number of particles needed to reach this bound. Note that for a given ε, M_χ is mostly linear in the number k of non-empty bins; the second, nonlinear term becomes negligible as k increases. The term $z_{1-\delta}$ is based on the parameter δ. It represents the upper $1-\delta$ quantile of the standard normal distribution. The values of $z_{1-\delta}$ for typical values of δ are readily available in standard statistical tables.

The algorithm generates new particles until their number M exceeds M_χ and a user-defined minimum $M_{\chi_{\min}}$. As can be seen, the threshold M_χ serves as a moving target for M. The more samples M are generated, the more bins k in the histogram are non-empty, and the higher the threshold M_χ.

In practice, the algorithm terminates based on the following reasoning. In the early stages of sampling, k increases with almost every new sample since virtually all bins are empty. This increase in k results in an increase in the threshold M_χ. However, over time, more and more bins are non-empty and M_χ increases only occasionally. Since M increases with each new sample, M will finally reach M_χ and sampling is stopped. When this happens depends on the belief. The more widespread the particles, the more bins are filled and the higher the threshold M_χ. During tracking, KLD-sampling generates less samples since the particles are concentrated on a small number of different bins. It should be noted that the histogram has no impact on the particle distribution itself. Its only purpose is to measure the complexity, or volume, of the belief. The grid is discarded at the end of each particle filter iteration.

Figure 8.18 shows the sample set sizes during a typical global localization run using KLD-sampling. The figure shows graphs when using a robot's laser range-finder (solid line) or ultrasound sensors (dashed line). In both cases, the algorithm chooses a large number of samples during the initial phase of global localization. Once the robot is localized, the number of particles drops to a much lower level (less than 1% of the initial number of particles). When and how fast this transition from global localization to position tracking happens depends on the type of the environment and the accuracy of the sensors. In this example, the higher accuracy of the laser range-finder is reflected by an earlier transition to a lower level.

Figure 8.19 shows a comparison between the approximation error of KLD-