Here atan2 is the common extension of the arcus tangens of y/x extended to the $\Re^2$ (most programming languages provide an implementation of this function):

(5.22) $$\text{atan2}(y,x) = \begin{cases} \text{atan}(y/x) & \text{if } x > 0 \\ \text{sign(y)}\ (\pi - \text{atan}(|\text{y}/\text{x}|)) & \text{if } x < 0 \\ 0 & \text{if } x = y = 0 \\ \text{sign(y)}\ \pi/2 & \text{if } x = 0, y \neq 0 \end{cases}$$

Since we assume that the robot follows a circular trajectory, the translational distance between x_t and x_{t-1} along this circle is

(5.23) $$\Delta\text{dist} = r^* \cdot \Delta\theta$$

From Δdist and $\Delta\theta$, it is now easy to compute the velocities $\hat{v}$ and $\hat{\omega}$:

(5.24) $$\hat{u}_t = \begin{pmatrix} \hat{v} \\ \hat{\omega} \end{pmatrix} = \Delta t^{-1} \begin{pmatrix} \Delta\text{dist} \\ \Delta\theta \end{pmatrix}$$

The rotational velocity $\hat{\gamma}$ needed to achieve the final heading θ' of the robot in $(x'y')$ within Δt can be determined according to (5.14) as:

(5.25) $$\hat{\gamma} = \Delta t^{-1}(\theta' - \theta) - \hat{\omega}$$

The *motion error* is the deviation of $\hat{u}_t$ and $\hat{\gamma}$ from the commanded velocity $u_t = (v\ \ \omega)^T$ and $\gamma = 0$, as defined in Equations (5.24) and (5.25).

(5.26) $$v_{\text{err}} = v - \hat{v}$$

(5.27) $$\omega_{\text{err}} = \omega - \hat{\omega}$$

(5.28) $$\gamma_{\text{err}} = \hat{\gamma}$$

Under our error model, specified in Equations (5.10), and (5.15), these errors have the following probabilities:

(5.29) $$\varepsilon_{\alpha_1 v^2 + \alpha_2 \omega^2}(v_{\text{err}})$$

(5.30) $$\varepsilon_{\alpha_3 v^2 + \alpha_4 \omega^2}(\omega_{\text{err}})$$

(5.31) $$\varepsilon_{\alpha_5 v^2 + \alpha_6 \omega^2}(\gamma_{\text{err}})$$

where ε_{b^2} denotes a zero-mean error variable with variance b^2, as before. Since we assume independence between the different sources of error, the desired probability $p(x_t \mid u_t, x_{t-1})$ is the product of these individual errors:

(5.32) $$p(x_t \mid u_t, x_{t-1}) = \varepsilon_{\alpha_1 v^2 + \alpha_2 \omega^2}(v_{\text{err}}) \cdot \varepsilon_{\alpha_3 v^2 + \alpha_4 \omega^2}(\omega_{\text{err}}) \cdot \varepsilon_{\alpha_5 v^2 + \alpha_6 \omega^2}(\gamma_{\text{err}})$$