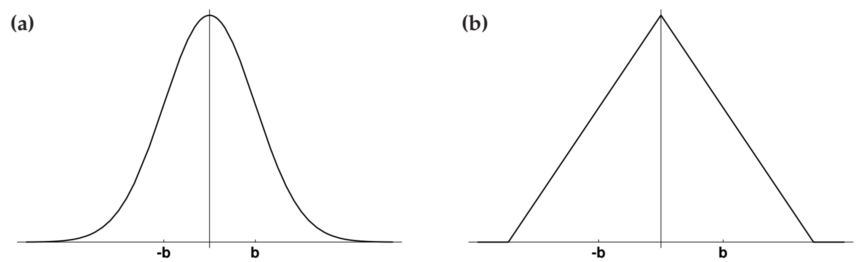

Figure 5.6 Probability density functions with variance b^2: (a) Normal distribution, (b) triangular distribution.

Two common choices for the error ε_{b^2} are the normal and the triangular distribution.

NORMAL DISTRIBUTION

The *normal distribution* with zero mean and variance b^2 is given by the density function

$$\varepsilon_{b^2}(a) = \frac{1}{\sqrt{2\pi\, b^2}}\, e^{-\frac{1}{2}\frac{a^2}{b^2}} \tag{5.11}$$

Figure 5.6a shows the density function of a normal distribution with variance b^2. Normal distributions are commonly used to model noise in continuous stochastic processes. Its support, which is the set of points a with $p(a) > 0$, is $\Re$.

TRIANGULAR DISTRIBUTION

The density of a *triangular distribution* with zero mean and variance b^2 is given by

$$\varepsilon_{b^2}(a) = \max\left\{0, \frac{1}{\sqrt{6}\,b} - \frac{|a|}{6\,b^2}\right\} \tag{5.12}$$

which is non-zero only in $(-\sqrt{6}b; \sqrt{6}b)$. As Figure 5.6b suggests, the density resembles the shape of a symmetric triangle—hence the name.

A better model of the actual pose $x_t = (x' \;\; y' \;\; \theta')^T$ after executing the motion command $u_t = (v \;\; \omega)^T$ at $x_{t-1} = (x \;\; y \;\; \theta)^T$ is thus

$$\begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} + \begin{pmatrix} -\frac{\hat{v}}{\hat{\omega}}\sin\theta + \frac{\hat{v}}{\hat{\omega}}\sin(\theta + \hat{\omega}\Delta t) \\ \frac{\hat{v}}{\hat{\omega}}\cos\theta - \frac{\hat{v}}{\hat{\omega}}\cos(\theta + \hat{\omega}\Delta t) \\ \hat{\omega}\Delta t \end{pmatrix} \tag{5.13}$$

This equation is obtained by substituting the commanded velocity $u_t = (v \;\; \omega)^T$ with the noisy motion $(\hat{v} \;\; \hat{\omega})^T$ in (5.9). However, this model is still not very realistic, for reasons discussed in turn.