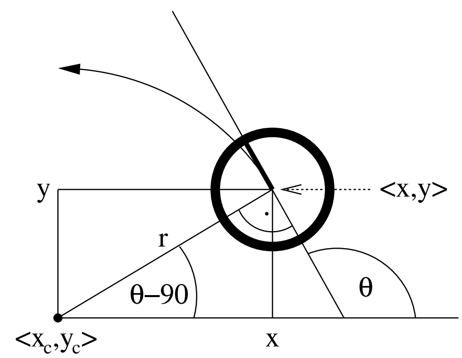

Figure 5.5 Motion carried out by a noise-free robot moving with constant velocities v and ω and starting at $(x\ \ y\ \ \theta)^T$.

Before turning to the probabilistic case, let us begin by stating the kinematics for an ideal, noise-free robot. Let $u_t = (v\ \ \omega)^T$ denote the control at time t. If both velocities are kept at a fixed value for the entire time interval $(t-1, t]$, the robot moves on a circle with radius

$$r = \left|\frac{v}{\omega}\right| \tag{5.5}$$

This follows from the general relationship between the translational and rotational velocities v and ω for an arbitrary object moving on a circular trajectory with radius r:

$$v = \omega \cdot r \tag{5.6}$$

Equation (5.5) encompasses the case where the robot does not turn at all (i.e., $\omega = 0$), in which case the robot moves on a straight line. A straight line corresponds to a circle with infinite radius, hence we note that r may be infinite.

Let $x_{t-1} = (x, y, \theta)^T$ be the initial pose of the robot, and suppose we keep the velocity constant at $(v\ \ \omega)^T$ for some time Δt. As one easily shows, the center of the circle is at

$$x_c = x - \frac{v}{\omega}\sin\theta \tag{5.7}$$

$$y_c = y + \frac{v}{\omega}\cos\theta \tag{5.8}$$