1: **Algorithm Discrete_Bayes_filter($\{p_{k,t-1}\}, u_t, z_t$):**
2: *for all* k *do*
3: $\bar{p}_{k,t} = \sum_i p(X_t = x_k \mid u_t, X_{t-1} = x_i)\, p_{i,t-1}$
4: $p_{k,t} = \eta\, p(z_t \mid X_t = x_k)\, \bar{p}_{k,t}$
5: *endfor*
6: *return* $\{p_{k,t}\}$

Table 4.1 The discrete Bayes filter. Here x_i, x_k denote individual states.

with a finite sum. The variables x_i and x_k denote individual states, of which there may only be finitely many. The belief at time t is an assignment of a probability to each state x_k, denoted $p_{k,t}$. Thus, the input to the algorithm is a discrete probability distribution $\{p_{k,t-1}\}$, along with the most recent control u_t and measurement z_t. Line 3 calculates the prediction, the belief for the new state based on the control alone. This prediction is then updated in line 4, so as to incorporate the measurement. The discrete Bayes filter algorithm is popular in many areas of signal processing, where it is often referred to as the forward pass of a *hidden Markov model*, or *HMM*.

HIDDEN MARKOV MODEL

4.1.2 Continuous State

Of particular interest will be the use of discrete Bayes filters as an approximate inference tool for *continuous* state spaces. As noted above, such filters are called *histogram filters*. Figure 4.1 illustrates how a histogram filter represents a random variable and its nonlinear transform. Shown there is the projection of a histogrammed Gaussian through a nonlinear function. The original Gaussian distribution possesses 10 bins. So does the projected probability distribution, but in two of the resulting bins the probability is so close to zero that they cannot be seen in this figure. Figure 4.1 also shows the correct continuous distributions for comparison.

Histogram filters decompose a continuous state space into finitely many *bins*, or *regions*:

$$\mathrm{dom}(X_t) \quad = \quad \mathbf{x}_{1,t} \cup \mathbf{x}_{2,t} \cup \ldots \mathbf{x}_{K,t} \tag{4.1}$$