This Gaussian is the approximation of the desired sampling distribution (13.26) in FastSLAM 2.0. Obviously, this proposal distribution is quite a bit more involved than the much simpler one for FastSLAM 1.0 in Equation (13.12).

13.4.2 Updating the Observed Feature Estimate

Just like our first version of the FastSLAM algorithm, FastSLAM 2.0 updates the posterior over the feature estimates based on the measurement z_t and the sampled pose $x_t^{[k]}$. The estimates at time $t-1$ are once again represented by the mean $\mu_{j,t-1}^{[k]}$ and the covariance $\Sigma_{j,t-1}^{[k]}$. The updated estimates are $\mu_{j,t}^{[k]}$ and $\Sigma_{j,t}^{[k]}$. The nature of the update depends on whether or not a feature j was observed at time t. For $j \neq c_t$, we already established in Equation (13.4) that the posterior over the feature remains unchanged. This implies that instead of updating the estimated, we merely have to copy it.

For the observed feature $j = c_t$, the situation is more intricate. Equation (13.5) already stated the posterior over observed features. Here we repeat it with the particle index k:

$$\begin{aligned} (13.43) \quad & p(m_{c_t} \mid x_t^{[k]}, c_{1:t}, z_{1:t}) \\ &= \eta \underbrace{p(z_t \mid m_{c_t}, x_t^{[k]}, c_t)}_{\sim \mathcal{N}(z_t; h(m_{c_t}, x_t^{[k]}), Q_t)} \underbrace{p(m_{c_t} \mid x_{1:t-1}^{[k]}, z_{1:t-1}, c_{1:t-1})}_{\sim \mathcal{N}(m_{c_t}; \mu_{c_t,t-1}^{[k]}, \Sigma_{c_t,t-1}^{[k]})} \end{aligned}$$

As in (13.27), the nonlinearity of h causes this posterior to be non-Gaussian, which is at odds with FastSLAM 2.0's Gaussian representation for feature estimates. Luckily, the exact same linearization as above provides the solution:

$$(13.44) \quad h(m_{c_t}, x_t) \approx \hat{z}_t^{[k]} + H_m(m_{c_t} - \mu_{c_t,t-1}^{[k]})$$

Notice that x_t is not a free variable here, hence we can omit the third term in (13.28). This approximation renders the probability (13.43) Gaussian in the target variable m_{c_t}:

$$\begin{aligned} (13.45) \quad & p(m_{c_t} \mid x_t^{[k]}, c_{1:t}, z_{1:t}) \\ &= \eta \exp\Big\{ -\tfrac{1}{2}(z_t - \hat{z}_t^{[k]} - H_m(m_{c_t} - \mu_{c_t,t-1}^{[k]}))^T Q_t^{-1} \\ &\qquad (z_t - \hat{z}_t^{[k]} - H_m(m_{c_t} - \mu_{c_t,t-1}^{[k]})) \\ &\qquad -\tfrac{1}{2}(m_{c_t} - \mu_{c_t,t-1}^{[k]})^T \Sigma_{c_t,t-1}^{[k]-1}(m_{c_t} - \mu_{c_t,t-1}^{[k]}) \Big\} \end{aligned}$$