

$$\begin{aligned}
&= \eta^{[k]} p(z_t | x_t, x_{1:t-1}^{[k]}, u_{1:t}, z_{1:t-1}, c_{1:t}) p(x_t | x_{1:t-1}^{[k]}, u_{1:t}, z_{1:t-1}, c_{1:t}) \\
\stackrel{\text{Markov}}{=} &\eta^{[k]} p(z_t | x_t, x_{1:t-1}^{[k]}, u_{1:t}, z_{1:t-1}, c_{1:t}) p(x_t | x_{t-1}^{[k]}, u_t) \\
&= \eta^{[k]} \int p(z_t | m_{c_t}, x_t, x_{1:t-1}^{[k]}, u_{1:t}, z_{1:t-1}, c_{1:t}) \\
&\quad p(m_{c_t} | x_t, x_{1:t-1}^{[k]}, u_{1:t}, z_{1:t-1}, c_{1:t}) dm_{c_t} p(x_t | x_{t-1}^{[k]}, u_t) \\
\stackrel{\text{Markov}}{=} &\eta^{[k]} \int \underbrace{p(z_t | m_{c_t}, x_t, c_t)}_{\sim \mathcal{N}(z_t; h(m_{c_t}, x_t), Q_t)} \underbrace{p(m_{c_t} | x_{1:t-1}^{[k]}, z_{1:t-1}, c_{1:t-1})}_{\sim \mathcal{N}(m_{c_t}; \mu_{c_t, t-1}^{[k]}, \Sigma_{c_t, t-1}^{[k]})} dm_{c_t} \\
&\quad \underbrace{p(x_t | x_{t-1}^{[k]}, u_t)}_{\sim \mathcal{N}(x_t; g(x_{t-1}^{[k]}, u_t), R_t)}
\end{aligned}$$

This expression makes apparent that our sampling distribution is truly the convolution of two Gaussians multiplied by a third. In the general SLAM case, the sampling distribution possesses no closed form from which we could easily sample. The culprit is the function h : If it were linear, this probability would be Gaussian, a fact that shall become more obvious below. In fact, not even the integral in (13.27) possesses a closed form solution. For this reason, sampling from the probability (13.27) is difficult.

This observation motivates the replacement of h by a linear approximation. As common in this book, this approximation is obtained through a first order Taylor expansion, given by the following linear function:

$$(13.28) \quad h(m_{c_t}, x_t) \approx \hat{z}_t^{[k]} + H_m(m_{c_t} - \mu_{c_t, t-1}^{[k]}) + H_x(x_t - \hat{x}_t^{[k]})$$

Here we use the following abbreviations:

$$(13.29) \quad \hat{z}_t^{[k]} = h(\mu_{c_t, t-1}^{[k]}, \hat{x}_t^{[k]})$$

$$(13.30) \quad \hat{x}_t^{[k]} = g(x_{t-1}^{[k]}, u_t)$$

The matrices H_m and H_x are the Jacobians of h . They are the derivatives of h with respect to m_{c_t} and x_t , respectively, evaluated at the expected values of their arguments:

$$(13.31) \quad H_m = \nabla_{m_{c_t}} h(m_{c_t}, x_t) \Big|_{x_t = \hat{x}_t^{[k]}, m_{c_t} = \mu_{c_t, t-1}^{[k]}}$$

$$(13.32) \quad H_x = \nabla_{x_t} h(m_{c_t}, x_t) \Big|_{x_t = \hat{x}_t^{[k]}, m_{c_t} = \mu_{c_t, t-1}^{[k]}}$$

Under this approximation, the desired sampling distribution (13.27) is a Gaussian with the following parameters:

$$(13.33) \quad \Sigma_{x_t}^{[k]} = \left[H_x^T Q_t^{[k]-1} H_x + R_t^{-1} \right]^{-1}$$