given by the quotient of the target and the proposal distribution:

$$w_t^{[k]} \;=\; \frac{\text{target distribution}}{\text{proposal distribution}} \;=\; \frac{p(x_{1:t}^{[k]} \mid z_{1:t}, u_{1:t}, c_{1:t})}{p(x_{1:t}^{[k]} \mid z_{1:t-1}, u_{1:t}, c_{1:t-1})} \;=\; \eta\, p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}, u_{1:t}, c_{1:t}) \tag{13.21}$$

The last transformation is a direct consequence of the following transformation of the numerator in (13.21):

$$\begin{aligned} & p(x_{1:t}^{[k]} \mid z_{1:t}, u_{1:t}, c_{1:t}) \\ &= \eta\, p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}, u_{1:t}, c_{1:t})\; p(x_{1:t}^{[k]} \mid z_{1:t-1}, u_{1:t}, c_{1:t}) \\ &= \eta\, p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}, u_{1:t}, c_{1:t})\; p(x_{1:t}^{[k]} \mid z_{1:t-1}, u_{1:t}, c_{1:t-1}) \end{aligned} \tag{13.22}$$

To calculate the probability $p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}, u_{1:t}, c_{1:t})$ in (13.21), it will be necessary to transform it further. In particular, this probability is equivalent to the following integration, where we once again omit variables irrelevant to the prediction of sensor measurements:

$$\begin{aligned} & w_t^{[k]} \\ &= \eta \int p(z_t \mid m_{c_t}, x_{1:t}^{[k]}, z_{1:t-1}, u_{1:t}, c_{1:t})\; p(m_{c_t} \mid x_{1:t}^{[k]}, z_{1:t-1}, u_{1:t}, c_{1:t})\; dm_{c_t} \\ &= \eta \int p(z_t \mid m_{c_t}, x_t^{[k]}, c_t)\; \underbrace{p(m_{c_t} \mid x_{1:t-1}^{[k]}, z_{1:t-1}, c_{1:t-1})}_{\sim \mathcal{N}((m_{c_t}; \mu_{c_t,t-1}^{[k]}, \Sigma_{c_t,t-1}^{[k]})}\; dm_{c_t} \end{aligned} \tag{13.23}$$

Here $\mathcal{N}(x; \mu, \Sigma)$ denotes a Gaussian distribution over the variable x with mean μ and covariance Σ.

The integration in (13.24) involves the estimate of the observed feature location at time t and the measurement model. To calculate (13.24) in closed form, FastSLAM employs the very same linear approximation used in the measurement update in Step 2. In particular, the importance factor is given by

$$w_t^{[k]} \;\approx\; \eta\, |2\pi Q_t^{[k]}|^{-\frac{1}{2}}\; \exp\left\{ -\tfrac{1}{2}(z_t - \hat{z}_t^{[k]})^T\, Q_t^{[k]-1} (z_t - \hat{z}_t^{[k]}) \right\} \tag{13.24}$$