1. The state transition probability $p(x_t \mid u_t, x_{t-1})$ must be a *linear* function in its arguments with added Gaussian noise. This is expressed by the following equation:

$$x_t = A_t x_{t-1} + B_t u_t + \varepsilon_t \tag{3.2}$$

Here x_t and x_{t-1} are state vectors, and u_t is the control vector at time t. In our notation, both of these vectors are vertical vectors. They are of the form

$$x_t = \begin{pmatrix} x_{1,t} \\ x_{2,t} \\ \vdots \\ x_{n,t} \end{pmatrix} \quad \text{and} \quad u_t = \begin{pmatrix} u_{1,t} \\ u_{2,t} \\ \vdots \\ u_{m,t} \end{pmatrix} \tag{3.3}$$

A_t and B_t are matrices. A_t is a square matrix of size $n \times n$, where n is the dimension of the state vector x_t. B_t is of size $n \times m$, with m being the dimension of the control vector u_t. By multiplying the state and control vector with the matrices A_t and B_t, respectively, the state transition function becomes *linear* in its arguments. Thus, Kalman filters assume linear system dynamics.

The random variable ε_t in (3.2) is a Gaussian random vector that models the uncertainty introduced by the state transition. It is of the same dimension as the state vector. Its mean is zero, and its covariance will be denoted R_t. A state transition probability of the form (3.2) is called a *linear Gaussian*, to reflect the fact that it is linear in its arguments with additive Gaussian noise. Technically, one may also include a constant additive term in (3.2), which is here omitted since it plays no role in the material to come.

Equation (3.4) defines the state transition probability $p(x_t \mid u_t, x_{t-1})$. This probability is obtained by plugging Equation (3.2) into the definition of the multivariate normal distribution (3.1). The mean of the posterior state is given by $A_t x_{t-1} + B_t u_t$ and the covariance by R_t:

$$\begin{aligned} p(x_t \mid u_t, x_{t-1}) &= \det(2\pi R_t)^{-\frac{1}{2}} \\ &\quad \exp\left\{-\tfrac{1}{2}(x_t - A_t x_{t-1} - B_t u_t)^T R_t^{-1}(x_t - A_t x_{t-1} - B_t u_t)\right\} \end{aligned} \tag{3.4}$$