calculated as follows:

$$
\begin{aligned}
G_t^{-1} &= (I + F_x^T \; \Delta \; F_x)^{-1} \\
&= (I \underbrace{-F_x^T \; I \; F_x + F_x^T \; I \; F_x}_{=0} + F_x^T \; \Delta \; F_x)^{-1} \\
&= (I - F_x^T \; I \; F_x + F_x^T \; (I + \Delta) \; F_x)^{-1} \\
&= I - F_x^T \; I \; F_x + F_x^T \; (I + \Delta)^{-1} \; F_x \\
&= I + \underbrace{F_x^T \; [(I + \Delta)^{-1} - I] \; F_x}_{\Psi_t} \\
&= I + \Psi_t
\end{aligned}
\tag{12.12}
$$

By analogy, we get for the transpose $[G_t^T]^{-1} = (I + F_x^T \; \Delta^T \; F_x)^{-1} = I + \Psi_t^T$. Here the matrix Ψ_t is only non-zero for elements that correspond to the robot pose. It is zero for all features in the map, and hence can be computed in constant time. This gives us for our desired matrix Φ_t the following expression:

$$
\begin{aligned}
\Phi_t &= (I + \Psi_t^T) \; \Omega_{t-1} \; (I + \Psi_t) \\
&= \Omega_{t-1} + \underbrace{\Psi_t^T \; \Omega_{t-1} + \Omega_{t-1} \; \Psi_t + \Psi_t^T \; \Omega_{t-1} \; \Psi_t}_{\lambda_t} \\
&= \Omega_{t-1} + \lambda_t
\end{aligned}
\tag{12.13}
$$

where Ψ_t is zero except for the sub-matrix corresponding to the robot pose. Since Ω_{t-1} is sparse, λ_t is zero except for a finite number of elements, which correspond to active map features and the robot pose.

Hence, Φ_t can be computed from Ω_{t-1} in constant time, assuming that Ω_{t-1} is sparse. Equations (12.11) through (12.13) are equivalent to lines 5 through 9 in Table 12.2, which proves the correctness of the information matrix update in **SEIF_motion_update**.

Finally, we show a similar result for the information vector. From (12.2) we obtain

$$
\bar{\mu}_t = \mu_{t-1} + F_x^T \; \delta_t
\tag{12.14}
$$

This implies for the information vector:

$$
\begin{aligned}
\bar{\xi}_t &= \bar{\Omega}_t \; (\Omega_{t-1}^{-1} \; \xi_{t-1} + F_x^T \; \delta_t) \\
&= \bar{\Omega}_t \; \Omega_{t-1}^{-1} \; \xi_{t-1} + \bar{\Omega}_t \; F_x^T \; \delta_t \\
&= (\bar{\Omega}_t + \Omega_{t-1} - \Omega_{t-1} + \Phi_t - \Phi_t) \; \Omega_{t-1}^{-1} \; \xi_{t-1} + \bar{\Omega}_t \; F_x^T \; \delta_t \\
&= (\bar{\Omega}_t \; \underbrace{-\Phi_t + \Phi_t}_{=0} \; \underbrace{-\Omega_{t-1} + \Omega_{t-1}}_{=0}) \; \Omega_{t-1}^{-1} \; \xi_{t-1} + \bar{\Omega}_t \; F_x^T \; \delta_t
\end{aligned}
\tag{12.15}
$$