

The derivation of the measurement update is similar to the one in Chapter 7.4. In particular, we are given the following measurement model

$$(10.17) \quad z_t^i = \underbrace{\begin{pmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ \text{atan2}(m_{j,y} - y, m_{j,x} - x) - \theta \\ m_{j,s} \end{pmatrix}}_{h(y_t, j)} + \mathcal{N}\left(0, \underbrace{\begin{pmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \sigma_\phi^2 & 0 \\ 0 & 0 & \sigma_s^2 \end{pmatrix}}_{Q_t}\right)$$

Here x , y , and θ denotes the pose of the robot, i is the index of an individual landmark observation in z_t , and $j = c_t^i$ is the index of the observed landmark at time t . The variable r denotes the range to a landmark, ϕ is the bearing to a landmark, and s the landmark signature; the terms σ_r , σ_ϕ , and σ_s are the corresponding measurement noise covariances.

This expression is approximated by the linear function

$$(10.18) \quad h(y_t, j) \approx h(\bar{\mu}_t, j) + H_t^i (y_t - \bar{\mu}_t)$$

Here H_t^i is the derivative of h with respect to the full state vector y_t . Since h depends only on two elements of that state vector, the robot pose x_t and the location of the j -th landmark m_j , the derivative factors into a low-dimensional Jacobian h_t^i and a matrix $F_{x,j}$, which maps h_t^i into a matrix of the dimension of the full state vector:

$$(10.19) \quad H_t^i = h_t^i F_{x,j}$$

Here h_t^i is the Jacobian of the function $h(y_t, j)$ at $\bar{\mu}_t$, calculated with respect to the state variables x_t and m_j :

$$(10.20) \quad h_t^i = \begin{pmatrix} \frac{\bar{\mu}_{t,x} - \bar{\mu}_{j,x}}{q_t} & \frac{\bar{\mu}_{t,y} - \bar{\mu}_{j,y}}{q_t} & 0 & \frac{\bar{\mu}_{j,x} - \bar{\mu}_{t,x}}{q_t} & \frac{\bar{\mu}_{j,y} - \bar{\mu}_{t,y}}{q_t} & 0 \\ \frac{\sqrt{q_t}}{\bar{\mu}_{j,y} - \bar{\mu}_{t,y}} & \frac{\sqrt{q_t}}{\bar{\mu}_{t,x} - \bar{\mu}_{j,x}} & -1 & \frac{\sqrt{q_t}}{\bar{\mu}_{t,y} - \bar{\mu}_{j,y}} & \frac{\sqrt{q_t}}{\bar{\mu}_{j,x} - \bar{\mu}_{t,x}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

The scalar $q_t = (\bar{\mu}_{j,x} - \bar{\mu}_{t,x})^2 + (\bar{\mu}_{j,y} - \bar{\mu}_{t,y})^2$, and as before, $j = c_t^i$ is the landmark that corresponds to the measurement z_t^i . The matrix $F_{x,j}$ is of dimension $6 \times (3N + 3)$. It maps the low-dimensional matrix h_t^i into a matrix