The map posterior factors into a map prior and a measurement likelihood (c.f., Equation (9.11)):

$$\log p(m \mid z_{1:t}, x_{1:t}) \;=\; \text{const.} \;+\; \log p(z_{1:t} \mid x_{1:t}, m) \;+\; \log p(m) \tag{9.21}$$

The log-likelihood $\log p(z_{1:t} \mid x_{1:t}, m)$ decomposes into a sum of individual measurement log-likelihoods:

$$\log p(z_{1:t} \mid x_{1:t}, m) \;=\; \sum \log p(z_t \mid x_t, m) \tag{9.22}$$

Further, the log-prior also decomposes. To see, we note that the prior probability of any map m is given by the following product:

$$\begin{aligned} p(m) &= \prod_i p(\mathbf{m})^{\mathbf{m}_i} \; (1 - p(\mathbf{m}))^{1-\mathbf{m}_i} \\ &= (1 - p(\mathbf{m}))^N \prod_i p(\mathbf{m})^{\mathbf{m}_i} \; (1 - p(\mathbf{m}))^{-\mathbf{m}_i} \\ &= \eta \prod_i p(\mathbf{m})^{\mathbf{m}_i} \; (1 - p(\mathbf{m}))^{-\mathbf{m}_i} \end{aligned} \tag{9.23}$$

Here $p(\mathbf{m})$ is the prior probability of occupancy (e.g., $p(\mathbf{m}) = 0.5$), and N is the number of grid cells in the map. The expression $(1 - p(\mathbf{m}))^N$ is simply a constant, which is replaced by our generic symbol η as usual.

This implies for the log version of the prior:

$$\begin{aligned} \log p(m) &= \text{const.} + \sum_i \mathbf{m}_i \; \log p(\mathbf{m}) \; - \mathbf{m}_i \; \log(1 - p(\mathbf{m})) \\ &= \text{const.} + \sum_i \mathbf{m}_i \; \log \frac{p(\mathbf{m})}{1 - p(\mathbf{m})} \\ &= \text{const.} + \sum_i \mathbf{m}_i \; l_0 \end{aligned} \tag{9.24}$$

The constant l_0 is adopted from (9.7). The term N $\log(1 - p(\mathbf{m}_i))$ is obviously independent of the map. Hence it suffices to optimize the remaining expression and the data log-likelihood:

$$m^* \;=\; \underset{m}{\text{argmax}} \sum_t \log p(z_t \mid x_t, m) + l_0 \; \sum_i \mathbf{m}_i \tag{9.25}$$

A hill-climbing algorithm for maximizing this log-probability is provided in Table 9.3. This algorithm starts with the all-free map (line 2). It "flips" the occupancy value of a grid cell when such a flip increases the likelihood of the data (lines 4-6). For this algorithm it is essential that the prior of occupancy