$$p(Z_t = \textbf{sense\_closed} \mid X_t = \textbf{is\_open}) \quad = \quad 0.4$$

and

$$\begin{aligned} p(Z_t = \textbf{sense\_open} \mid X_t = \textbf{is\_closed}) &= 0.2 \\ p(Z_t = \textbf{sense\_closed} \mid X_t = \textbf{is\_closed}) &= 0.8 \end{aligned}$$

These probabilities suggest that the robot's sensors are relatively reliable in detecting a *closed* door, in that the error probability is 0.2. However, when the door is open, it has a 0.4 probability of an erroneous measurement.

Finally, let us assume the robot uses its manipulator to push the door open. If the door is already open, it will remain open. If it is closed, the robot has a 0.8 chance that it will be open afterwards:

$$\begin{aligned} p(X_t = \textbf{is\_open} \mid U_t = \textbf{push}, X_{t-1} = \textbf{is\_open}) &= 1 \\ p(X_t = \textbf{is\_closed} \mid U_t = \textbf{push}, X_{t-1} = \textbf{is\_open}) &= 0 \\ p(X_t = \textbf{is\_open} \mid U_t = \textbf{push}, X_{t-1} = \textbf{is\_closed}) &= 0.8 \\ p(X_t = \textbf{is\_closed} \mid U_t = \textbf{push}, X_{t-1} = \textbf{is\_closed}) &= 0.2 \end{aligned}$$

It can also choose not to use its manipulator, in which case the state of the world does not change. This is stated by the following conditional probabilities:

$$\begin{aligned} p(X_t = \textbf{is\_open} \mid U_t = \textbf{do\_nothing}, X_{t-1} = \textbf{is\_open}) &= 1 \\ p(X_t = \textbf{is\_closed} \mid U_t = \textbf{do\_nothing}, X_{t-1} = \textbf{is\_open}) &= 0 \\ p(X_t = \textbf{is\_open} \mid U_t = \textbf{do\_nothing}, X_{t-1} = \textbf{is\_closed}) &= 0 \\ p(X_t = \textbf{is\_closed} \mid U_t = \textbf{do\_nothing}, X_{t-1} = \textbf{is\_closed}) &= 1 \end{aligned}$$

Suppose at time $t = 1$, the robot takes no control action but it senses an open door. The resulting posterior belief is calculated by the Bayes filter using the prior belief $bel(X_0)$, the control $u_1 = \textbf{do\_nothing}$, and the measurement **sense_open** as input. Since the state space is finite, the integral in line 3 turns into a finite sum:

$$\begin{aligned} &\overline{bel}(x_1) \\ &\quad = \int p(x_1 \mid u_1, x_0)\, bel(x_0)\, dx_0 \\ &\quad = \sum_{x_0} p(x_1 \mid u_1, x_0)\, bel(x_0) \end{aligned}$$