$$= \int p(z_t^i \mid x_t, c_t^i, m) \, \overline{bel}(x_t) \, dx_t$$

The left term in the final integral is the measurement likelihood assuming knowledge of the robot location x_t. This likelihood is given by a Gaussian with mean at the measurement that is expected at location x_t. This measurement, denoted $\hat{z}_t^i$, is provided by the measurement function h. The covariance of the Gaussian is given by the measurement noise Q_t.

(7.18) $$p(z_t^i \mid x_t, c_t^i, m) \sim \mathcal{N}(z_t^i;\ h(x_t, c_t^i, m),\ Q_t) \approx \mathcal{N}(z_t^i;\ h(\bar{\mu}_t, c_t^i, m) + H_t\,(x_t - \bar{\mu}_t),\ Q_t)$$

(7.18) follows by applying our Taylor expansion (7.13) to h. Plugging this equation back into (7.17), and replacing $\overline{bel}(x_t)$ by its Gaussian form, we get the following measurement likelihood:

(7.19) $$p(z_t^i \mid c_{1:t}, m, z_{1:t-1}, u_{1:t}) \approx \mathcal{N}(z_t^i;\ h(\bar{\mu}_t, c_t^i, m) + H_t\,(x_t - \bar{\mu}_t),\ Q_t) \otimes \mathcal{N}(x_t;\ \bar{\mu}_t, \bar{\Sigma}_t)$$

where $\otimes$ denotes the familiar convolution over the variable x_t. This equation reveals that the likelihood function is a convolution of two Gaussians; one representing the measurement noise, the other representing the state uncertainty. We already encountered integrals of this form in Chapter 3.2, where we derived the motion update of the Kalman filter and the EKF. The closed-form solution to this integral is derived completely analogously to those derivations. In particular, the Gaussian defined by (7.19) has mean $h(\bar{\mu}_t, c_t^i, m)$ and covariance $H_t\ \bar{\Sigma}_t\ H_t^T + Q_t$. Thus, we have under our linear approximation the following expression for the measurement likelihood:

(7.20) $$p(z_t^i \mid c_{1:t}, m, z_{1:t-1}, u_{1:t}) \sim \mathcal{N}(z_t^i;\ h(\bar{\mu}_t, c_t^i, m),\ H_t\,\bar{\Sigma}_t\,H_t^T + Q_t)$$

That is,

(7.21) $$p(z_t^i \mid c_{1:t}, m, z_{1:t-1}, u_{1:t}) = \eta\ \exp\left\{-\frac{1}{2}(z_t^i - h(\bar{\mu}_t, c_t^i, m)\,)^T\ [H_t\,\bar{\Sigma}_t\,H_t^T + Q_t]^{-1}\ (z_t^i - h(\bar{\mu}_t, c_t^i, m)\,)\right\}$$

By replacing the mean and covariance of this expression by $\hat{z}_t^i$ and S_t, respectively, we get line 21 of the EKF algorithm in Table 7.2.

The EKF localization algorithm can now easily be modified to accommodate outliers. The standard approach is to only accept landmarks for which the likelihood passes a threshold test. This is generally a good idea: Gaussians fall off exponentially, and a single outlier can have a huge effect on the pose estimate. In practice, thresholding adds an important layer of robustness to the algorithm without which EKF localization tends to be brittle.