The function $g(u_t, \mu_{t-1})$ is simply obtained by replacing the exact state x_{t-1}—which we do not know—by our expectation μ_{t-1}—which we know. The Jacobian G_t is the derivative of the function g with respect to x_{t-1} evaluated at u_t and μ_{t-1}:

(7.8)
$$G_t \;=\; \frac{\partial g(u_t, \mu_{t-1})}{\partial x_{t-1}} \;=\; \begin{pmatrix} \frac{\partial x'}{\partial \mu_{t-1,x}} & \frac{\partial x'}{\partial \mu_{t-1,y}} & \frac{\partial x'}{\partial \mu_{t-1,\theta}} \\ \frac{\partial y'}{\partial \mu_{t-1,x}} & \frac{\partial y'}{\partial \mu_{t-1,y}} & \frac{\partial y'}{\partial \mu_{t-1,\theta}} \\ \frac{\partial \theta'}{\partial \mu_{t-1,x}} & \frac{\partial \theta'}{\partial \mu_{t-1,y}} & \frac{\partial \theta'}{\partial \mu_{t-1,\theta}} \end{pmatrix}$$

Here $\mu_{t-1} = (\mu_{t-1,x} \;\; \mu_{t-1,y} \;\; \mu_{t-1,\theta})^T$ denotes the mean estimate factored into its individual three values, and $\frac{\partial x'}{\partial \mu_{t-1,x}}$ is short for the derivative of g along the x' dimension, taken with respect to x at μ_{t-1}. Calculating these derivatives from Equation (7.6) gives us the following matrix:

(7.9)
$$G_t \;=\; \begin{pmatrix} 1 & 0 & \frac{v_t}{\omega_t}(-\cos\mu_{t-1,\theta} + \cos(\mu_{t-1,\theta} + \omega_t \Delta t)) \\ 0 & 1 & \frac{v_t}{\omega_t}(-\sin\mu_{t-1,\theta} + \sin(\mu_{t-1,\theta} + \omega_t \Delta t)) \\ 0 & 0 & 1 \end{pmatrix}$$

To derive the covariance of the additional motion noise, $\mathcal{N}(0, R_t)$, we first determine the covariance matrix M_t of the noise in *control space*. This follows directly from the motion model in Equation (7.5):

(7.10)
$$M_t \;=\; \begin{pmatrix} \alpha_1 v_t^2 + \alpha_2 \omega_t^2 & 0 \\ 0 & \alpha_3 v_t^2 + \alpha_4 \omega_t^2 \end{pmatrix}$$

The motion model in (7.6) requires this motion noise to be mapped into *state space*. The transformation from control space to state space is performed by another linear approximation. The Jacobian needed for this approximation, denoted V_t, is the derivative of the motion function g with respect to the motion parameters, evaluated at u_t and μ_{t-1}:

(7.11)
$$\begin{aligned} V_t \;&=\; \frac{\partial g(u_t, \mu_{t-1})}{\partial u_t} \\ &=\; \begin{pmatrix} \frac{\partial x'}{\partial v_t} & \frac{\partial x'}{\partial \omega_t} \\ \frac{\partial y'}{\partial v_t} & \frac{\partial y'}{\partial \omega_t} \\ \frac{\partial \theta'}{\partial v_t} & \frac{\partial \theta'}{\partial \omega_t} \end{pmatrix} \\ &=\; \begin{pmatrix} \frac{-\sin\theta + \sin(\theta + \omega_t \Delta t)}{\omega_t} & \frac{v_t(\sin\theta - \sin(\theta + \omega_t \Delta t))}{\omega_t^2} + \frac{v_t \cos(\theta + \omega_t \Delta t)\Delta t}{\omega_t} \\ \frac{\cos\theta - \cos(\theta + \omega_t \Delta t)}{\omega_t} & -\frac{v_t(\cos\theta - \cos(\theta + \omega_t \Delta t))}{\omega_t^2} + \frac{v_t \sin(\theta + \omega_t \Delta t)\Delta t}{\omega_t} \\ 0 & \Delta t \end{pmatrix} \end{aligned}$$

The multiplication $V_t \; M_t \; V_t^T$ then provides an approximate mapping between the motion noise in control space to the motion noise in state space.